

Logic Gates

Dr.Ahmed Bayoumi
Dr.Shady Elmashad

Objectives

- ▶ Identify the basic gates and describe the behavior of each
- ▶ Combine basic gates into circuits
- ▶ Describe the behavior of a gate using Boolean expressions, truth tables, and logic diagrams



Definition

- ▶ A **gate** is a device that performs a basic operation on electrical signals
- ▶ Gates are combined into **circuits** to perform more complicated tasks
- ▶ describing the behavior of gates and circuits by
 - ▶ Boolean expressions
 - ▶ logic diagrams
 - ▶ truth tables



Gates

six types of gates

- ▶ NOT
- ▶ AND
- ▶ OR
- ▶ XOR
- ▶ NAND
- ▶ NOR



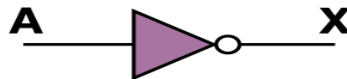
NOT Gate

- ▶ A NOT gate accepts one input value and produces one output value
- ▶ By definition, if the input value for a NOT gate is 0, the output value is 1, and if the input value is 1, the output is 0
- ▶ A NOT gate is sometimes referred to as an *inverter* because it inverts the input value

Boolean Expression

$$X = A'$$

Logic Diagram Symbol



Truth Table

A	X
0	1
1	0

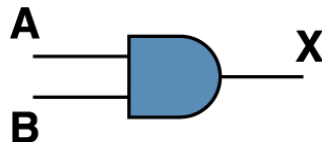
AND Gate

- ▶ An AND gate accepts two input signals
- ▶ If the two input values for an AND gate are both 1, the output is 1; otherwise, the output is 0

Boolean Expression

$$X = A \cdot B$$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

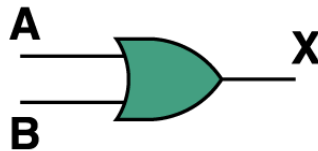
OR Gate

- ▶ If the two input values are both 0, the output value is 0; otherwise, the output is 1

Boolean Expression

$$X = A + B$$

Logic Diagram Symbol



Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

XOR Gate

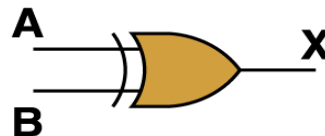
▶ XOR, or *exclusive* OR, gate

- ▶ An XOR gate produces 0 if its two inputs are the same, and a 1 otherwise
- ▶ Note the difference between the XOR gate and the OR gate; they differ only in one input situation
- ▶ When both input signals are 1, the OR gate produces a 1 and the XOR produces a 0

Boolean Expression

$$X = A \oplus B$$

Logic Diagram Symbol

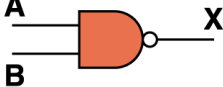


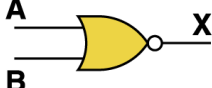
Truth Table

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

NAND and NOR Gates

- ▶ The NAND and NOR gates are essentially the opposite of the AND and OR gates, respectively

Boolean Expression	Logic Diagram Symbol	Truth Table															
$X = (A \cdot B)'$		<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	X	0	0	1	0	1	1	1	0	1	1	1	0
A	B	X															
0	0	1															
0	1	1															
1	0	1															
1	1	0															

Boolean Expression	Logic Diagram Symbol	Truth Table															
$X = (A + B)'$		<table><tr><th>A</th><th>B</th><th>X</th></tr><tr><td>0</td><td>0</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table>	A	B	X	0	0	1	0	1	0	1	0	0	1	1	0
A	B	X															
0	0	1															
0	1	0															
1	0	0															
1	1	0															

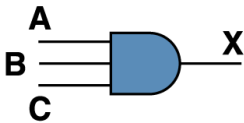
Review of Gate Processing

- ▶ A NOT gate inverts its single input value
- ▶ An AND gate produces 1 if both input values are 1
- ▶ An OR gate produces 1 if one or the other or both input values are 1
- ▶ An XOR gate produces 1 if one or the other (but not both) input values are 1
- ▶ A NAND gate produces the opposite results of an AND gate
- ▶ A NOR gate produces the opposite results of an OR gate



Gates with More Inputs

- ▶ Gates can be designed to accept three or more input values
- ▶ A three-input AND gate, for example, produces an output of 1 only if all input values are 1

Boolean Expression	Logic Diagram Symbol	Truth Table																																				
$X = A \cdot B \cdot C$		<table><tr><th>A</th><th>B</th><th>C</th><th>X</th></tr><tr><td>0</td><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>0</td><td>1</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td><td>0</td></tr><tr><td>1</td><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td><td>1</td></tr></table>	A	B	C	X	0	0	0	0	0	0	1	0	0	1	0	0	0	1	1	0	1	0	0	0	1	0	1	0	1	1	0	0	1	1	1	1
A	B	C	X																																			
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1	1	0	0																																			
1	1	1	1																																			

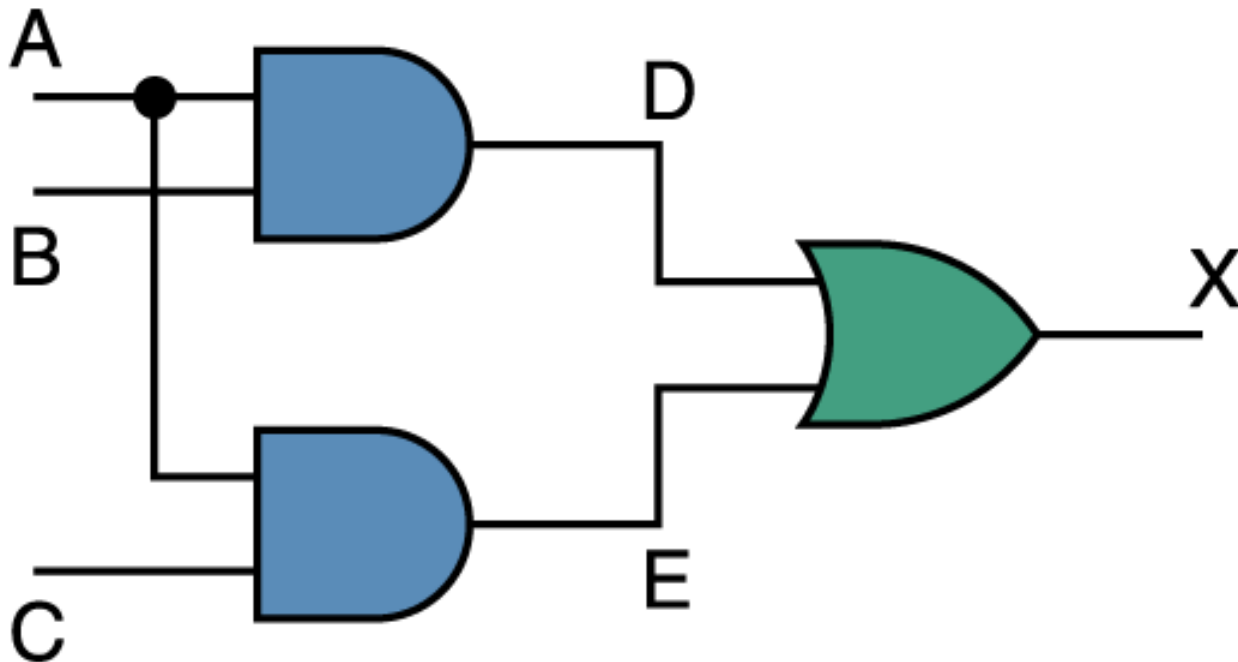
Circuits

- ▶ Two general categories
 - ▶ In a **combinational circuit**, the input values explicitly determine the output
 - ▶ In a **sequential circuit**, the output is a function of the input values as well as the existing state of the circuit
- ▶ As with gates, we can describe the operations of entire circuits using three notations
 - ▶ Boolean expressions
 - ▶ logic diagrams
 - ▶ truth tables



Combinational Circuits

- ▶ Gates are combined into circuits by using the output of one gate as the input for another



Combinational Circuits

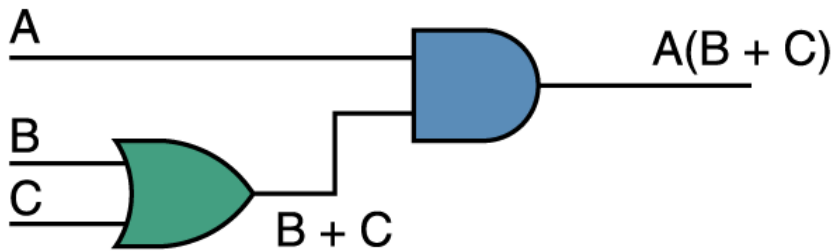
A	B	C	D	E	X
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	1	1
1	1	0	1	0	1
1	1	1	1	1	1

- ▶ Because there are three inputs to this circuit, eight rows are required to describe all possible input combinations
- ▶ This same circuit using Boolean algebra:

$$(AB + AC)$$

Now let's go the other way; let's take a Boolean expression and draw

- ▶ Consider the following Boolean expression: $A(B + C)$



A	B	C	B + C	A(B+C)
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	0	0
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

- Now compare the final result column in this truth table to the truth table for the previous example
 - They are identical

Properties of Boolean Algebra

Property	AND	OR
Commutative	$AB = BA$	$A + B = B + A$
Associative	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive	$A(B + C) = (AB) + (AC)$	$A + (BC) = (A + B)(A + C)$
Identity	$A1 = A$	$A + 0 = A$
Complement	$A(A') = 0$	$A + (A') = 1$
DeMorgan's law	$(AB)' = A' \text{ OR } B'$	$(A + B)' = A'B'$

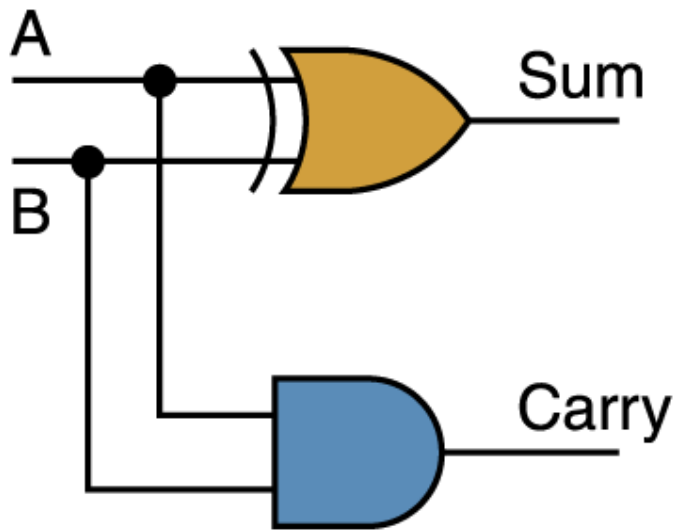


Adders

- ▶ At the digital logic level, addition is performed in binary
- ▶ Addition operations are carried out by special circuits called, appropriately, **adders**
- ▶ The result of adding two binary digits could produce a *carry value*
- ▶ Recall that $1 + 1 = 10$ in base two
- ▶ A circuit that computes the sum of two bits and produces the correct carry bit is called a **half adder**



Adders



► Circuit diagram representing a half adder

► Two Boolean expressions:

$$\text{sum} = A \oplus B$$

$$\text{carry} = AB$$

A	B	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

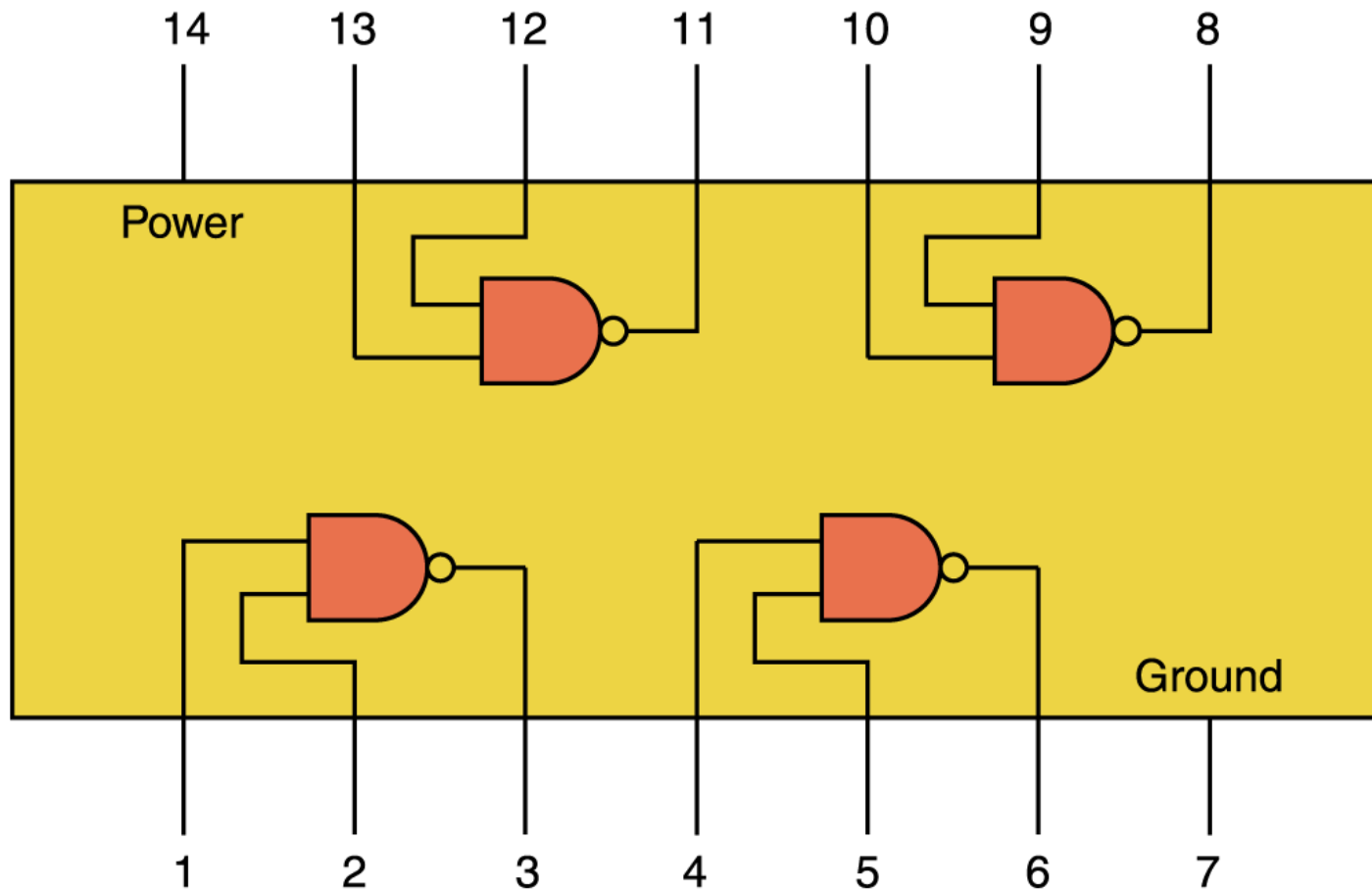
Integrated Circuits

- ▶ Integrated circuits (IC) are classified by the number of gates contained in them

Abbreviation	Name	Number of Gates
SSI	Small-Scale Integration	1 to 10
MSI	Medium-Scale Integration	10 to 100
LSI	Large-Scale Integration	100 to 100,000
VLSI	Very-Large-Scale Integration	more than 100,000



Integrated Circuits



CPU Chips

- ▶ The most important integrated circuit in any computer is the Central Processing Unit, or CPU
- ▶ Each CPU chip has a large number of pins through which essentially all communication in a computer system occurs

